On the rate of evaporation of water into a stream of dry air, humidified air and superheated steam, and the inversion temperature

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Abstract

The concept of inversion temperature was developed in order to assess the effectiveness of superheated steam as a drying agent when compared with dry or humid air above a given temperature. The inversion temperature can be defined in different ways, for different physical situations, thus leading to strong discrepancies between its reported numerical values. A clear understanding of such definitions and its consequences over the inversion temperature values is of major importance, as well as the way to estimate the inversion temperature of interest for a given practical evaporation process. Analytical expressions are proposed for some possible definitions of the inversion temperature. Special emphasis is devoted to the physical reasoning on the existence of the inversion temperature and how its value is affected by other variables such as humidity level, mass flow rate and geometry of the evaporative system.
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1. Introduction

The drying requirements of some sensitive materials (wood and wood products, ceramics, etc.) have led to evaluation of alternative drying agents such as superheated steam and air-superheated steam mixtures, particularly at high temperatures. The implications of the use of alternative drying agents upon final product quality, energy requirements and duration and economics of the operation have been investigated [1–3]. However, the complete understanding of the reasons for which one drying medium is more favourable than other has not been thoroughly accomplished.

The usual convective solid drying process relies upon moisture removal from a solid matrix and consequent humidification of a gas phase (generally, moist air). The common perception of the drying process states that air streams are more effective drying agents at low humidity and high temperature. However, this is true only below a given temperature, the so-called inversion temperature: at temperatures above the inversion temperature, highly humid air, or even pure superheated steam, are in fact more effective drying agents than dry air.

The knowledge of superheated steam as a drying agent comes from many years ago, as reported by Keey [4], but is essentially the work of Yoshida and Hyodo [5] that set the existence of the inversion temperature. The advantages of using a very humid drying agent from the viewpoint of product quality are well known, especially for foodstuffs and other natural products [6], even if some corrosion problems can be encountered.

The models usually used to find the inversion temperature consider a wet (liquid) surface, which releases unbounded water into a stream of humid air or superheated steam. Despite the apparent simplicity of the physical situation described, no strong agreement between the inversion temperatures calculated by several authors was found. Yoshida and Hyodo [5] reported an inversion temperature of near 170 °C, which was slightly affected by the mass flow rate of the drying agent. Using well known empirical correlations for the heat transfer Nusselt number, Nomura and Yodo [6] reported an

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Nomenclature

Ac	Ackermann correction factor
ASIT	adiabatic saturation inversion temperature
C	constant
c_p	constant pressure specific heat
D	diameter
EIT	effective inversion temperature
h	heat transfer coefficient
h_{fg}	latent heat of vaporization
L	length
LIT	local inversion temperature
m	exponent of the heat transfer correlation
$\dot{m}$	mass flow rate
$\dot{m}''$	mass flux
n	exponent of the heat transfer correlation
Nu	Nusselt number
P	pressure
Pr	Prandtl number
P_w	wetted perimeter
$\dot{q}''$	heat flux
Re	Reynolds number
RM	molar mass ratio
Sc	Schmidt number
Sh	Sherwood number
T	temperature

V	velocity
W	specific humidity (dry basis)
x	distance along the drying surface

Greek symbols

β	dimensionless auxiliary coefficient
χ	dimensionless steam mass flow rate
Δ	difference value
ρ	density

Superscript

h	specific humidity in wet basis

Subscripts

a	air
ad	adiabatic saturation conditions
atm	at the atmospheric pressure
ev	evaporation
in	inlet
s	saturation conditions
t	total pressure
v	vapour
x	at distance x over the drying surface
W	stream with inlet humidity W

inversion temperature near 260 °C, which is independent of the mass flow rate of the drying agent, if radiation heat transfer is not taken into account. Chow and Chung [7] obtained a laminar similarity solution of the complete mass-momentum-energy boundary layer equations, both for variable properties and for constant properties using the one-third rule, which lead to an inversion temperature near 250 °C. This work was complemented by Hasan et al. [8], where the laminar boundary layer evaporation of water, benzene and methanol from their moving wet surfaces into co-current streams of pure air, unsaturated mixtures of air and vapours, and superheated vapours were compared. For the air-water pair, the inversion temperature was found to be near 260 °C, which was slightly changed by changing the wet surface velocity. For the benzene-air pair and for the methanol-air pair, there is no inversion temperature within the operating temperature range of practical interest (100–350 °C). In a more recent work, Sheikholeslami and Watkinson [9] encountered an inversion temperature near 140 °C for the water–air pair, if the dryer was taken as an adiabatic saturation chamber, this temperature changing when varying the humidity level of the incoming drying agent. The recent work by Schwartze and Bröcker [10] is an attempt to solve some of the questions related with the existence and the existing discrepancy between the numerical values of the inversion temperature.

Since a strong discrepancy between the reported values of the inversion temperature for the most common water–air pair is observed, the main objective of this work is hence a contribution towards a more complete understanding of the physical reasons behind the definition, existence and dependency of the inversion temperature. From this point of view, this paper identifies different definitions used for the inversion temperature clarifying their physical meaning. The authors propose new relations for calculation of the inversion temperature illustrating its dependence upon several variables. The analysis is limited to the water–air pair at atmospheric pressure, although the concepts and definitions can be expanded to other pairs ant total pressures, provided their thermophysical properties and saturation pressure dependence upon temperature are known.

2. Heat and mass transfer analysis and modeling

The situation analysed herewith is sketched in Fig. 1, which represents steady state evaporation of water into a gaseous stream of dry air, moist air or superheated
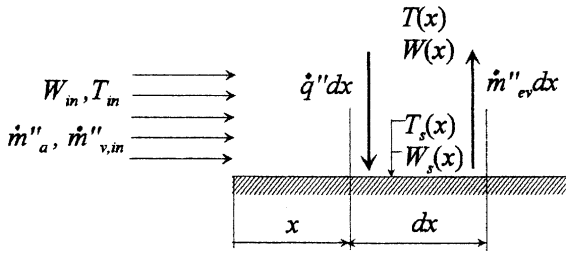
Fig. 1. Geometry and variables for the analysed situations.

steam along the liquid surface. Radiation heat transfer is neglected, even if in the present analysis moderate temperature levels are considered.

2.1. Local heat and mass transfer modeling

Over any element dx along the surface, the $\dot{q}''dx$ heat flow (by unit depth) reaching the surface is the difference between the heat transferred by convection from the gaseous phase and the enthalpy flow associated with the vapour flow $\dot{m}''_{ev}dx$ (also by unit depth) that is incorporated into the gaseous stream. At a given length x along the surface

$$\dot{q}'' = h(T - T_s)\left[\frac{\dot{m}''_{ev}c_{P,v}/h}{\exp(\dot{m}''_{ev}c_{P,v}/h) - 1}\right] \tag{1}$$

The term within square brackets is the *Ackermann correction factor*, referred simply as Ac, which accounts for simultaneous occurrence of convective heat and mass transfer [4].

For the situation presented in Fig. 1, as there is no heat transfer through the liquid layer, the heat reaching the liquid surface is integrally used to evaporate the water, that is

$$\dot{q}'' = \dot{m}''_{ev}h_{fg(T_s)} \tag{2}$$

The surface temperature, T_s, is the local wet bulb temperature under the *local* (T, W) gaseous stream conditions, which is obtained from Eqs. (1) and (2) as

$$T_s = T - \frac{1}{Ac}\frac{\dot{m}''_{ev}}{h}h_{fg(T_s)} \tag{3}$$

This is different of the strict wet bulb temperature, which is defined on the basis of a high mass flow rate stream whose conditions are not affected by the evaporation process. Assuming water vapour saturation at the surface, and ideal gas behaviour for air and vapour, W_s depends upon T_s through

$$W_s = RM\frac{P_{v,s(T_s)}}{P_t - P_{v,s(T_s)}} \tag{4}$$

where RM is the molar mass ratio, $RM = M_v/M_a$ (for the water–air pair, $RM = 0.62185$), and $P_{v,s(T_s)}$ is the saturation pressure of water vapor at temperature T_s.

The mass flux of water, the unique transferred component, is expressed as [4,11]

$$\dot{m}''_{ev} = \frac{h}{c_{P,W}}\left(\frac{Pr}{Sc}\right)^{(1-n)}RM\ln\left(\frac{W_s + RM}{W + RM}\right) \tag{5}$$

where $c_{P,W} = (c_{P,a} + Wc_{P,v})/(1 + W)$ is the *humid* specific heat. In the calculations presented here, the specific enthalpies are expressed as the product of constant average specific heats and temperature differences, which is an acceptable approach for the present purposes. Exponent $(1 - n)$ depends on the geometry and flow regime of the gaseous stream, n being the exponent of the Prandtl and Schmidt numbers in correlations of the type $Nu = CRe^mPr^n$ and $Sh = CRe^mSc^n$. In many situations of interest, $n = 1/3$.

It should be noted that when the drying agent is pure steam there is no mass transfer resistance, Eq. (5) does not apply, and the evaporation flux is controlled by the heat transfer flux reaching the liquid surface, being obtained from the single combination of Eqs. (1) and (2), the involved thermophysical properties being that of pure superheated steam. In this same case, Eq. (4) does not apply to obtain the interface temperature. This temperature is given directly by the vapour pressure–temperature relationship for the total pressure of the system. In order to unify the treatment, the pure superheated steam can (approximately) be treated as humid air with a vapour mass fraction of 99%, which corresponds to a humidity (dry basis) of 99 kg/kg.

2.2. Forward simulation of the evaporation process along a liquid surface

Evaporation taking place along the liquid surface is modeled step-by-step focusing on an element surface area of length Δx in the flow direction, as sketched in Fig. 1, when $dx \rightarrow \Delta x$, where mass and energy (enthalpy) balances are made. For each element one obtains that

$$(\dot{m}_a)_{x+\Delta x} = (\dot{m}_a)_x = \dot{m}_a \tag{6}$$

$$(W)_{x+\Delta x} = (W)_x + \frac{\dot{m}''_{ev}\Delta x P_w}{(\dot{m}_a + \dot{m}_v)_{in}}(1 + W_{in}) \tag{7}$$

$$(T)_{x+\Delta x} = (T)_x - [(W)_{x+\Delta x} - (W)_x]$$
$$\times \frac{h_{fg(T_s)} + c_{P,v}[(T)_x - T_s]}{c_{P,a} + (W)_{x+\Delta x}c_{P,v}} \tag{8}$$

In the foregoing expressions P_w is the wetted perimeter with liquid, the evaporation taking place through the $\Delta x P_w$ surface area in each element, and subscript 'in' refers to the origin of the evaporation surface, at $x = 0$. $(\dot{m}_a + \dot{m}_v)_{in}$ is the total mass flow rate at the system inlet.
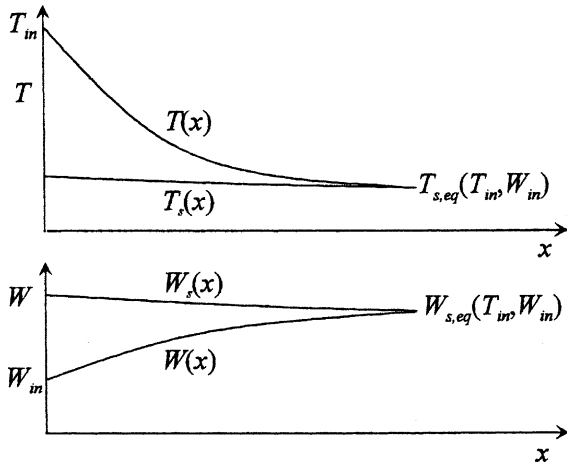
Fig. 2. Evolution of the drying variables along the evaporative surface.

Within each Δx element it is assumed that $T = (T)_x$ and $W = (W)_x$ for the evaluation of T_s, W_s, and $\dot{m}''_{ev}$, an acceptable practice for a sufficiently small increment Δx.

The obtained results are of the form presented graphically in Fig. 2. Along the drying surface, the local wet bulb temperature, $T_s(x)$, which is a function of the local gaseous stream conditions $[T(x), W(x)]$, decreases slightly as x increases, the same occurring with the saturation humidity at the surface (the maximum moisture content decreases as temperature decreases). As expected, the temperature of the gaseous stream $T(x)$ decreases and its humidity $W(x)$ increases as x increases.

For a given length L, measured away from the inlet, the overall evaporating mass flow rate can be obtained as $(\dot{m}_{ev})_{0-L} = \dot{m}_a(W_L - W_{in})$, or

$$(\dot{m}_{ev})_{0-L} = (\dot{m}_a + \dot{m}_v)_{in}\frac{W_L - W_{in}}{1 + W_{in}} \qquad (9)$$

If the gaseous stream is pure steam, the concept of humidity makes no sense, and the equations that apply are not Eqs. (7) and (8) but, respectively,

$$(\chi)_{x+\Delta x} = (\chi)_x + \frac{\dot{m}''_{ev}\Delta x P_w}{(\dot{m}_v)_{in}} \qquad (10)$$

$$(T)_{x+\Delta x} = (T)_x - \left[(\chi)_{x+\Delta x} - (\chi)_x\right]$$
$$\times \frac{h_{fg(T_{s,v})} + c_{P,v}\left[(T)_x - T_{s,v}\right]}{(\chi)_{x+\Delta x}c_{P,v}} \qquad (11)$$

where $\chi = \dot{m}_v/(\dot{m}_v)_{in}$ is the dimensionless steam mass flow rate, referred to the inlet steam mass flow rate $(\dot{m}_v)_{in}$, and $T_{s,v}$ is the saturation temperature of the pure steam for the imposed total pressure. If $P_t = P_{atm}$, $T_{s,v} = 100$ °C. In this case, for a given length L, measured away from the inlet, the overall evaporating mass flow rate can be obtained as

$$(\dot{m}_{ev})_{0-L} = (\dot{m}_v)_{in}(\chi_L - 1) \qquad (12)$$

noting that $\chi_{in} = 1$.

2.3. Adiabatic saturation conditions for a sufficiently long dryer

For a distance $L_{s,ad}$ away from the origin, the temperature and humidity stream conditions approach the surface conditions, reaching asymptotically the saturation conditions corresponding to the inlet conditions (T_{in}, W_{in}). The potentials for heat and mass transfer come to zero, and there is no more local heat and mass transfer. Naturally, more severe drying conditions would lead to shorter distances $L_{s,ad}$. The dryer approaches an adiabatic saturation chamber, but it is not a true adiabatic saturation chamber in the usual sense, as the evaporation temperature T_s is not constant along the overall evaporation surface. In this particular case, assuming constant specific heats and considering that the overall liquid surface is maintained at the uniform temperature T_s, the humidity changes from W_{in} to $(W)_L = W_s$ and the stream temperature changes from T_{in} to $(T)_L = T_s$. From Eq. (8) one obtains

$$(c_{P,a} + W_{in}c_{P,v})(T_{in} - T_s) = (W_s - W_{in})h_{fg(T_s)} \qquad (13)$$

that, together with Eq. (4), leads to the adiabatic saturation values T_s and W_s. These adiabatic saturation conditions are near the wet bulb saturation conditions, as the psychrometric coefficient for the water–air pair is close to the unit [11]. However, for the present calculations, wet bulb saturation conditions and adiabatic saturation conditions are different situations leading to slightly different values of T_s and W_s. Eq. (7) is not used in this case, as the particular aspects of the occurring mass transfer along the dryer are irrelevant; relevant is the fact that the adiabatic saturation conditions are reached. The overall evaporating mass flow rate can be obtained from Eq. (9) as

$$\dot{m}_{ev} = (\dot{m}_a + \dot{m}_v)_{in}\frac{W_s - W_{in}}{1 + W_{in}}$$
$$= (\dot{m}_a + \dot{m}_v)_{in}\frac{c_{P,a} + W_{in}c_{P,v}}{1 + W_{in}}\frac{T_{in} - T_s}{h_{fg(T_s)}} \qquad (14)$$

If the gaseous stream is pure steam, one obtains similarly from Eq. (11) that

$$c_{P,v}(T_{in} - T_{s,v}) = (\chi_s - 1)h_{fg(T_{s,v})} \qquad (15)$$

In this case, the overall evaporating mass flow rate can be obtained as

$$\dot{m}_{ev} = (\dot{m}_v)_{in}(\chi_s - 1) = (\dot{m}_v)_{in}\frac{c_{P,v}(T_{in} - T_{s,v})}{h_{fg(T_{s,v})}} \qquad (16)$$

3. Different inversion temperature definitions and numerical values

3.1. Local inversion temperature

The strict *local inversion temperature* is defined as the inlet temperature that corresponds to equal *local* evaporation rates from a small element surface area to two streams of dry air and pure superheated steam. It is referred as LIT_0, where the subscript 0 means that it is associated with a stream of dry air ($W_{in} = 0$). As evaporation takes place, the stream conditions change, and such a local inversion temperature makes sense only at the inlet section of the dryer, where the stream conditions remain unchanged.

This concept of local inversion temperature can be enlarged in order to consider also the situations when the local evaporation rates from a small element surface area to two streams of humidified air and pure superheated steam are equal, which is referred as LIT_W. The strict local inversion temperature LIT_0 is thus just a particular case of LIT_W when $W_{in} = 0$. Such definitions of the inversion temperature can be useful from a conceptual viewpoint, but not from a practical one.

To obtain the local evaporating mass flux, $\dot{m}''_{ev}$, which is evaluated from Eq. (5) or a combination of Eqs. (1) and (2) for pure steam, one needs to know the adequate correlation to obtain the convection heat transfer coefficient, h. This is a source of discrepancies for the inversion temperature values reported in the literature. For the turbulent flow in a circular duct (an internally humidified column, with unbound water) one can use the correlation [12]

$$h = 0.027 \frac{k}{D} Re_D^{0.8} Pr^{1/3} \left[1 + \left(\frac{D}{x} \right)^{2/3} \right] \tag{17}$$

where the square-bracketed term is a factor to consider the increase of the heat transfer coefficient in the developing region relative to the fully developed region. Over the fully developed region it is $D/x = 0$. The use of different correlations or different considerations of the developing region inevitably leads to different inversion temperature values. The validity of correlations of the type of Eq. (17) to evaluate the heat transfer coefficient was analysed by Schwartze and Bröker [10], by comparison with the experimental results of Yoshida and Hyodo [5], without the bracketed term (that is, without considering the heat transfer particularities of the developing region). They concluded that such correlation slightly underpredicts the experimental evaporating mass flow rates. This may be due to the non-consideration of the developing region, where the heat transfer is more intense than in the fully developed region. For $D/x < 1/60$, typical errors of less than 15% are associated with neglecting the developing region, and consid-

ering fully developed regime over all the length [12]. However, special care is needed when using this kind of correlations at the origin of the heat and mass transfer surface, due to its singularity when $x \to 0^+$. In order to avoid this special region and the inversion temperature dependence on the used treatment, it is considered here the heat transfer coefficient corresponding to the fully developed region over the overall length of the column.

For the flow over a flat plate, from a conceptual viewpoint the question is similar with the one occurring inside a duct. Only the local inversion temperature can be evaluated at the origin of the evaporative surface, as the inlet conditions remain constant there. The aforementioned singularity when $x \to 0^+$ is also present in this case [12]. As mentioned before, also in this case the local inversion temperature is not of significant practical usefulness.

Considering one case analysed by Yoshida and Hyodo [5], for a column of 0.029 m diameter, and a constant inlet mass flux of 18 200 kg/(m² h), one can obtain the local evaporating mass flux $(\dot{m}''_{ev})_W$ corresponding to a gaseous stream of humidity W, and the local evaporating mass flux $(\dot{m}''_{ev})_v$ corresponding to a pure superheated stream. The inlet temperature for which the two evaporating fluxes are equal is the local inversion temperature. The numerically obtained local mass fluxes are presented in Fig. 3a, and the associated local inversion temperatures are presented in Fig. 3b as function of the inlet humidity level. It is clear from Fig. 3a that the evaporative mass flux for the air stream is greater than that corresponding to highly humid air or even pure steam for lower temperature levels, this behaviour being inverted for higher temperature levels. It can be seen from Fig. 3b the strong dependency of the local inversion temperature on the inlet humidity level of the gaseous stream, the inversion temperature decreasing as the humidity level increases.

One can go one step further, by proposing an expression for the local inversion temperature from which could be analysed the influence of several variables and parameters over the numerical value of the local inversion temperature. Equating the evaporative mass fluxes $(\dot{m}''_{ev})_W$ given by Eq. (5) and $(\dot{m}''_{ev})_v$ obtained from the combination of Eqs. (1) and (2), one obtains

$$LIT_W = T_{s,v} + \left(\frac{V_W}{V_v} \right)^m \left(\frac{RM}{Ac_v} \right) \left(\frac{h_{fg(T_s)}}{c_{P,v}} \right) \left(\frac{\rho_W}{\rho_v} \right)$$
$$\times \left(\frac{Pr_v}{Sc_W} \right)^{(1-n)} \left(\frac{Sc_W}{Sc_v} \right)^{(1-m)} \ln \left(\frac{W_s + RM}{W + RM} \right) \tag{18}$$

where $W = W_{in}$, noting that the inlet temperature T_{in} for which $(\dot{m}''_{ev})_W = (\dot{m}''_{ev})_v$ is the local inversion temperature LIT_W. For $P_t = P_{atm}$, $T_{s,v} = 100$ °C. T_s and W_s are the local wet bulb conditions for a humid stream of humidity W, obtained from Eqs. (3) and (4).
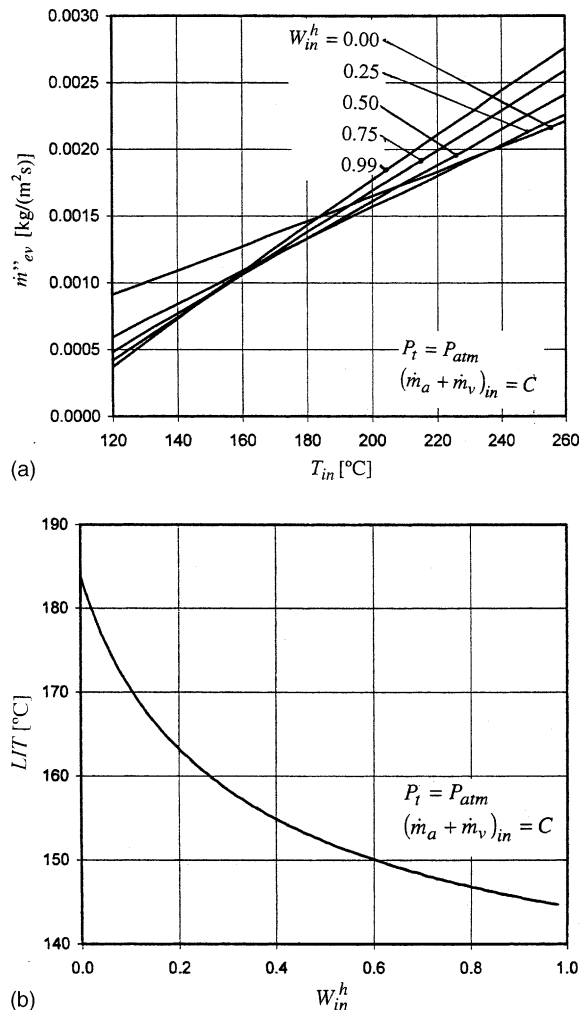
Fig. 3. Local inversion temperature for a particular case, under constant inlet mass flow rate: (a) Evaporative mass flux for some inlet humidity levels as function of the inlet temperature; and (b) The local inversion temperature dependence on the inlet humidity level.

From Eq. (18) one concludes that the local inversion temperature is always above the saturation temperature of the pure steam, $T_{s,v}$, by the long product present in that equation. The product $(\rho_W/\rho_v)(Pr_v/Sc_W)^{(1-n)} \times (Sc_W/Sc_v)^{(1-m)}$ is only dependent of the humidity level W, decreasing as W increases. Exponents m and n also affect the calculated local inversion temperature, correlations with different values for m and n leading to different values of LIT_W. The involved velocity ratio is $(V_W/V_v)^m = 1$ if the inlet velocity of the different streams is the same (constant volumetric flow rate), or $(V_W/V_v)^m = (\rho_v/\rho_W)^m < 1$ if the inlet mass flow rate of the different streams is the same. One concludes thus that the local inversion temperature corresponding to

constant inlet mass flow rate is lower than that corresponding to constant inlet velocity. This aspect has been showed by Schwartze and Bröcker [10] by results analysis. Similar conclusions can be easily extracted for other inlet mass flow rate ratios.

It is to be noted that Eq. (18) is not an explicit equation for LIT_W. Effectively, on the right-hand side, Ac_v depends on $\dot{m}''_{ev}$, which depends on the absolute velocity V_v and on the inlet temperature $T_{in} = LIT_W$. By their own turn, T_s and W_s are obtained from Eqs. (3) and (4), being thus equally dependent on W, LIT_W and $\dot{m}''_{ev}$. As humidity level increases, so increase T_s and W_s, but the remaining factors on the right-hand side of Eq. (18) lead to a decrease of LIT_W. If W increases strongly, the gaseous stream approaches pure superheated steam, the logarithm in Eq. (18) vanishes, thus giving $LIT_W \to T_{s,v}$.

To the authors' knowledge, Eq. (18) is the first proposed expression for the local inversion temperature. Even if it is an implicit expression for LIT_W, it is a very useful tool to evaluate its numerical value as well as to explore how its value is affected by several variables and parameters.

In what concerns the physical reasons for the existence of the local inversion temperature, there are some aspects that can be pointed out. The first is the absence of a mass transfer resistance when the gaseous stream is composed by pure steam, the local evaporating mass flux being controlled only by the local convection heat transfer flux. As the humidity level increases so decreases the mass transfer resistance. When the gaseous stream is air or humidified air, there are a mass transfer resistance and a heat transfer resistance. If the temperature is low enough, the temperature difference is small, thus giving rise to small heat and mass transfer fluxes controlled essentially by heat transfer. As the temperature increases, the temperature difference also increases, the same occurring with the heat and mass transfer fluxes. However, the maximum heat transfer flux is linked to the maximum mass transfer flux, which corresponds to the maximum humidity difference $W_s - W$, reached with the highest saturation humidity at the liquid interface, increasing this humidity difference as the inlet humidity increases. For air or humid air streams, the heat and mass transfer problem is thus mainly mass transfer controlled, due to the existing mass transfer resistance.

In terms of thermophysical properties, the Prandtel number of the superheatd steam is greater than that of air, the inverse occurring with the cinematic viscosities and the thermal conductivities. This represents a 5% heat transfer advantage of steam over air, for equal inlet velocities, and a 50% heat transfer advantage of steam over air for the same inlet mass flow rates of steam and air. For this last case, the strong heat transfer advantage of steam over air pulls down the local inversion temperature. Additionally, as humidity increases, so in-

creases the interface temperature, T_s, resulting in lower values of the latent heat, $h_{fg(T_s)}$.

3.2. Effective inversion temperature

More useful is the *effective inversion temperature*, defined as the inlet temperature that corresponds to equal *global* evaporation rates from a given surface area to two streams of humidified air and pure superheated steam, referred as EIT_W. This is the inversion temperature of practical usefulness, above which the global evaporation rate is greater when the drying agent is the superheated steam than when it is the humidified air, and the superheated steam becomes more effective as drying agent.

The evolution of the variables along the evaporative surface is obtained by the aforementioned forward practice, the global evaporation rate being obtained by integration of the local evaporation rate along the considered length, or from Eqs. (9) and (12). The inlet temperature T_{in} leading to equal mass flow rates, $(\dot{m}_{ev})_W = (\dot{m}_{ev})_v$, or

$$\frac{W_L - W_{in}}{1 + W_{in}} = \frac{(\dot{m}_v)_{in}}{(\dot{m}_a + \dot{m}_v)_{in}}(\chi_L - 1) \qquad (19)$$

is the effective inversion temperature, EIT_W. The effective inversion temperature depends of the local heat and mass transfer coefficients, and of the extension of the considered liquid surface, and thus of the particular geometry and stream flow conditions. Different systems under different conditions lead to different numerical values of the effective inversion temperature. It should be stressed that $EIT_W \to LIT_W$ when $L \to 0$.

For constant inlet mass flow rate it is $(\dot{m}_v)_{in}/(\dot{m}_a + \dot{m}_v)_{in} = 1$, and for constant inlet velocity it is $(\dot{m}_v)_{in}/(\dot{m}_a + \dot{m}_v)_{in} = \rho_v/\rho_W < 1$. However, it should be noted that for different conditions one has different local heat and mass transfer fluxes, leading to different global mass flow rates for a given length, and thus different numerical values of the effective inversion temperature. The influence of the involved variables and parameters over the effective inversion temperature is similar to that referred when analysing the local inversion temperature, added of the extension of the evaporative surface.

For the same situation considered before, that is, for a wetted column of internal diameter 0.029 m, and a constant inlet mass flux of 18 200 kg/(m² h), the numerically evaluated effective inversion temperature corresponding to some humidity levels is presented in Fig. 4 as function of the considered length, with and without consideration of the developing region. Correlation (17) is used even if the interface temperature slightly decreases with x, along the evaporative surface. It is observed from Fig. 4 that, for a given inlet humidity level,
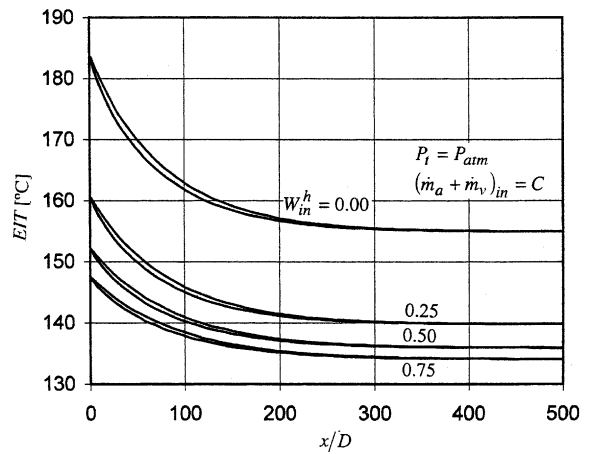

Fig. 4. Effective inversion temperature for some inlet humidity levels as function of the considered length along the evaporative surface.

the effective inversion temperature decreases with the considered length. An equilibrium value is asymptotically reached for a sufficiently long length, corresponding to the saturation conditions of the gaseous stream, with no further heat and mass transfer occurring. More severe drying conditions leads to shorter lengths needed to reach the equilibrium conditions. Greater inlet humidity levels lead to lower inversion temperatures for a given considered length. The strong dependency of the effective inversion temperature on the particular aspects of the heat and mass transfer processes can also be observed in Fig. 4, the consideration of the developing region leading always to lower effective inversion temperatures than when the particular heat and mass transfer aspects of this region are not considered. It is also observed from Fig. 4 that $EIT_W \to LIT_W$ when $x \to 0$. The consideration of a constant inlet velocity will lead to higher effective inversion temperatures, essentially by the reasons explained before when analysing the local inversion temperature.

### 3.3. The inversion temperature based upon the adiabatic saturation process

For the conditions considered in Section 2.3, one can define another inversion temperature, the *adiabatic saturation inversion temperature* for a given inlet humidity level W, $ASIT_W$. This is defined as the inlet temperature that leads to equal evaporative mass flow rates for a stream of humidified air and for a stream of pure superheated steam, such that both streams are saturated at the dryer exit, and all the evaporation processes takes place at the (saturation) exit temperature and humidity. This corresponds to a dryer that can be modeled as an adiabatic saturation chamber. This is somewhat different

from the situation considered before, for a sufficiently long length, because the interface temperature was not constant along the evaporative surface, as illustrated in Fig. 2.

The adiabatic saturation inversion temperature is obtained as the inlet temperature leading to equal evaporative mass flow rates given by Eqs. (14) and (16). The numerically evaluated dimensionless mass flow rates $\dot{m}_{ev}/(\dot{m}_a + \dot{m}_v)_{in} = (W_s - W_{in})/(1 + W_{in})$ and $\dot{m}_{ev}/(\dot{m}_v)_{in} = (\chi_s - 1)$, for constant inlet mass flow rates, $(\dot{m}_a + \dot{m}_v)_{in} = (\dot{m}_v)_{in}$, are presented in Fig. 5a as function of the inlet
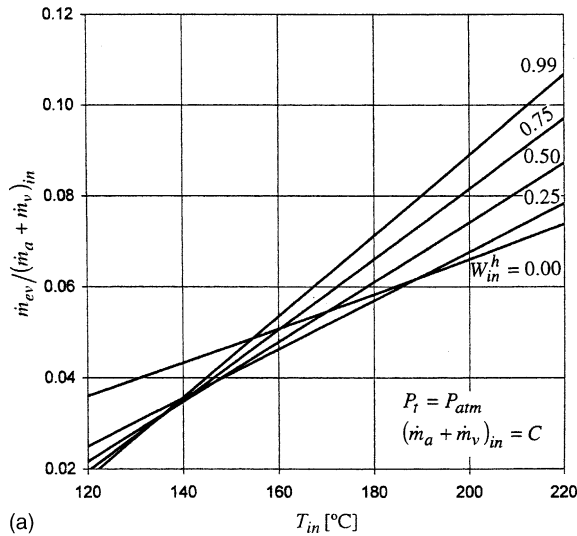
(a)
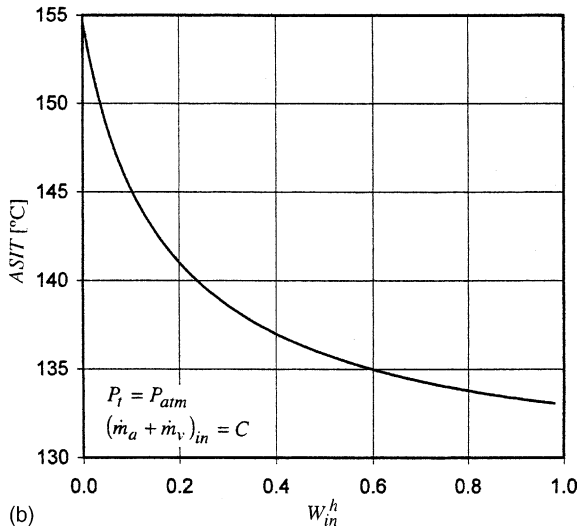
(b)

Fig. 5. Adiabatic saturation inversion temperature for the water–air pair, under constant inlet mass flow rate: (a) dimensionless evaporative mass flux for some inlet humidity levels as function of the inlet temperature; and (b) the adiabatic saturation inversion temperature dependence on the inlet humidity level.

temperature for some inlet humidity levels. Such results are essentially the same as reported by Sheikholeslami and Watkinson [9]. The corresponding adiabatic saturation inversion temperatures are presented in Fig. 5b as function of the inlet humidity level. Fig. 5a is similar to Fig. 3a, and Fig. 5b is similar to Fig. 3b. One can conclude that, for a given inlet humidity level, the local inversion temperature (for the particular conditions considered) is higher than the adiabatic saturation inversion temperature. However, it must be retained that the local inversion temperature is dependent of the particular heat and mass transfer processes, and that the adiabatic saturation inversion temperature is independent of such heat and mass transfer processes.

For this situation one can go another step further, proposing an expression for the adiabatic saturation inversion temperature. From Eqs. (14) and (16) one obtains that

$$\text{ASIT}_W = T_{s,v} + \frac{\beta}{1-\beta}(T_{s,v} - T_s);$$

$$\beta = \frac{(\dot{m}_a + \dot{m}_v)_{in}}{(\dot{m}_v)_{in}} \frac{c_{P,a} + W_{in}c_{P,v}}{1 + W_{in}} \frac{1}{c_{P,v}} \frac{h_{fg(T_{s,v})}}{h_{fg(T_s)}} \quad (20)$$

as the inlet temperature is $T_{in} = \text{ASIT}_W$. T_s is the adiabatic saturation temperature for a stream of humid air of inlet humidity W_{in} and inlet temperature $T_{in} = \text{ASIT}_W$, reason why Eq. (20) is not an explicit expression for ASIT_W. For $P_t = P_{atm}$ it is $T_{s,v} = 100\ °C$.

$T_{s,v} > T_s$, $h_{fg(T_{s,v})} < h_{fg(T_s)}$, $c_{P,a} < c_{P,v}$ and $(c_{P,a} + W_{in}c_{P,v})/[(1 + W_{in})c_{P,v}] < 1$. For the situation of constant inlet mass flow rate it is $(\dot{m}_a + \dot{m}_v)_{in}/(\dot{m}_v)_{in} = 1$, and for the situation of constant inlet velocity it is $(\dot{m}_a + \dot{m}_v)_{in}/(\dot{m}_v)_{in} = (\rho_w/\rho_v) > 1$ thus leading to greater values of β. The inversion temperature must be always above $T_{s,v}$ as at the inlet one has superheated steam, and Eq. (20) makes sense only for $\beta < 1$. A mass flow ratio $(\dot{m}_a + \dot{m}_v)_{in}/(\dot{m}_v)_{in}$ leading to $\beta > 1$ corresponds to a situation where the evaporative mass flow rates cannot be equal, and the adiabatic saturation inversion temperature does not exist. In fact, as $T_s < T_{s,v}$ it is $(T_{in} - T_{s,v}) < (T_{in} - T_s)$, and from Eqs. (14) and (16) it can be easily concluded that $(\dot{m}''_{ev})_v = (\dot{m}''_{ev})_a$ only if $\beta < 1$.

As the humidity level increases so increases T_s and decreases the temperature difference $T_{s,v} - T_s$, the same occurring with the adiabatic saturation inversion temperature. For highly humid streams $T_s \to T_{s,v}$ and $\beta \to 1$, the product $[\beta/(1-\beta)](T_{s,v} - T_s)$ vanishes and $\text{ASIT} \to T_{s,v}$. As β increases also ASIT_W increases, the constant inlet velocity situation leading to a higher adiabatic saturation inversion temperatures than the constant inlet mass flow rate situation.

To the authors' knowledge, Eq. (20) is the first proposed expression for the adiabatic saturation inversion temperature defined herewith. It is a very useful tool to

evaluate its numerical value, as well as to explore how this inversion temperature is affected by the involved variables and parameters.

The physical reasons for the existence of the adiabatic saturation inversion temperature should not rely on the heat and mass transfer aspects, as they are irrelevant in this case. To simplify the analysis, it is made only for the situations of dry air and pure superheated steam with constant inlet mass flow rate $(\dot{m}_a + \dot{m}_v)_{in} = (\dot{m}_v)_{in} = \dot{m}_{in}$. From Eqs. (14) and (16) one obtains that $(\dot{m}_{ev})_a = \dot{m}_{in} c_{P,a} (T_{in} - T_s)/h_{fg(T_s)}$ and $(\dot{m}_{ev})_v = \dot{m}_{in} c_{P,v} \times (T_{in} - T_{s,v})/h_{fg(T_{s,v})}$, with $c_{P,v} \approx 2c_{P,a}$. As $T_s < T_{s,v}$ it is $h_{fg(T_{s,v})} < h_{fg(T_s)}$. For low temperature levels, $T_{in} - T_s$ is considerably higher than $T_{in} - T_{s,v}$, and $(\dot{m}_{ev})_a > (\dot{m}_{ev})_v$. For high temperature levels, the temperature difference $T_{in} - T_{s,v}$ increases and when multiplied by $c_{P,v}$ results in a product higher than $c_{P,a}(T_{in} - T_s)$. The fact that $c_{P,v} \approx 2c_{P,a}$ is present in Fig. 5a, the slope of the line corresponding to the pure superheated steam being nearly the double of the one of the line corresponding to dry air. Thus, the principal physical reason for the existence of the adiabatic saturation inversion temperature can be attributed to the higher specific heat of the steam relative to that of dry air. Given the greater specific heat of steam, more humidity can be incorporated into the steam stream with lower decreasing of its temperature, towards the saturation temperature. In the light of this arguments, the conjugation of the specific heat and the mass flow rate results into a physical explanation of the adiabatic saturation inversion temperature in terms of the heat capacities of the involved streams.

4. Conclusions

Streams of highly humidified air or even superheated steam are increasingly used as drying agents, with new needs of a complete understanding of the occurring evaporative processes in terms of heat and mass transfer. Of special interest is the inlet temperature above which the stream of steam is a more effective drying agent than the stream of humidified air, known as the inversion temperature.

Depending on the definition of the inversion temperature, as well as on the particular process under analysis, one can obtain different (or even very different) numerical values of the inversion temperature. For a given process this represents a serious problem, bringing additional difficulties to find the inversion temperature, the heating costs, the mass/volume flow rates, the equipment size, the production rate, etc. Specially important is the clarification brought about some possible definitions of the inversion temperature, and the reasons why different definitions and practical situations lead to different (or even very different) numerical results. This work results in the proposition of a systematic tool for

the analysis of such systems, and for the finding of the evaporative mass flow rates and of the inversion temperatures. The numerical values of the local inversion temperature and of the effective inversion temperature strongly depend on the local heat and mass transfer processes taking place, and thus of the quality of the correlations used to evaluate the local convection heat transfer coefficient.

Another expected important contribution of this work is the proposition of analytical expressions for some possible definitions of the inversion temperature. Such expressions are very useful tools both to evaluate the numerical value of the inversion temperature and to analyse the influence of several variables and parameters over this value, namely the inlet mass flow rate ratio, the stream velocity, the inlet humidity level, the total pressure, and the steam and air thermophysical properties.

The presented results refer only to the water–air pair, at the atmospheric pressure, for constant inlet mass flow rates, but the proposed method of analysis is general. It can be easily applied to other operating conditions, to other pairs of substances, as well as to other total pressures than the atmospheric pressure.

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